PATH INTEGRAL FORMULATION OF LIGHT TRANSPORT

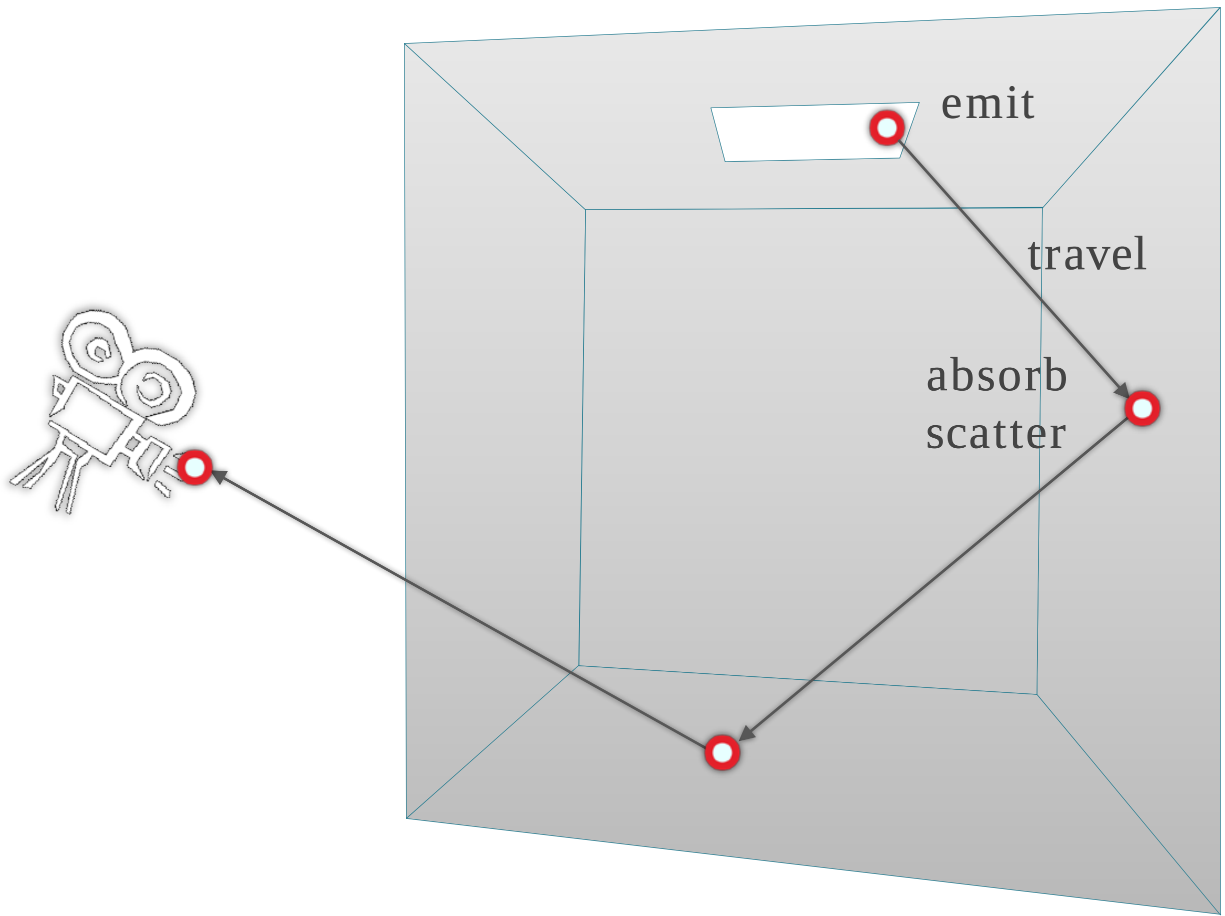
Jaroslav Křivánek

Charles University in Prague

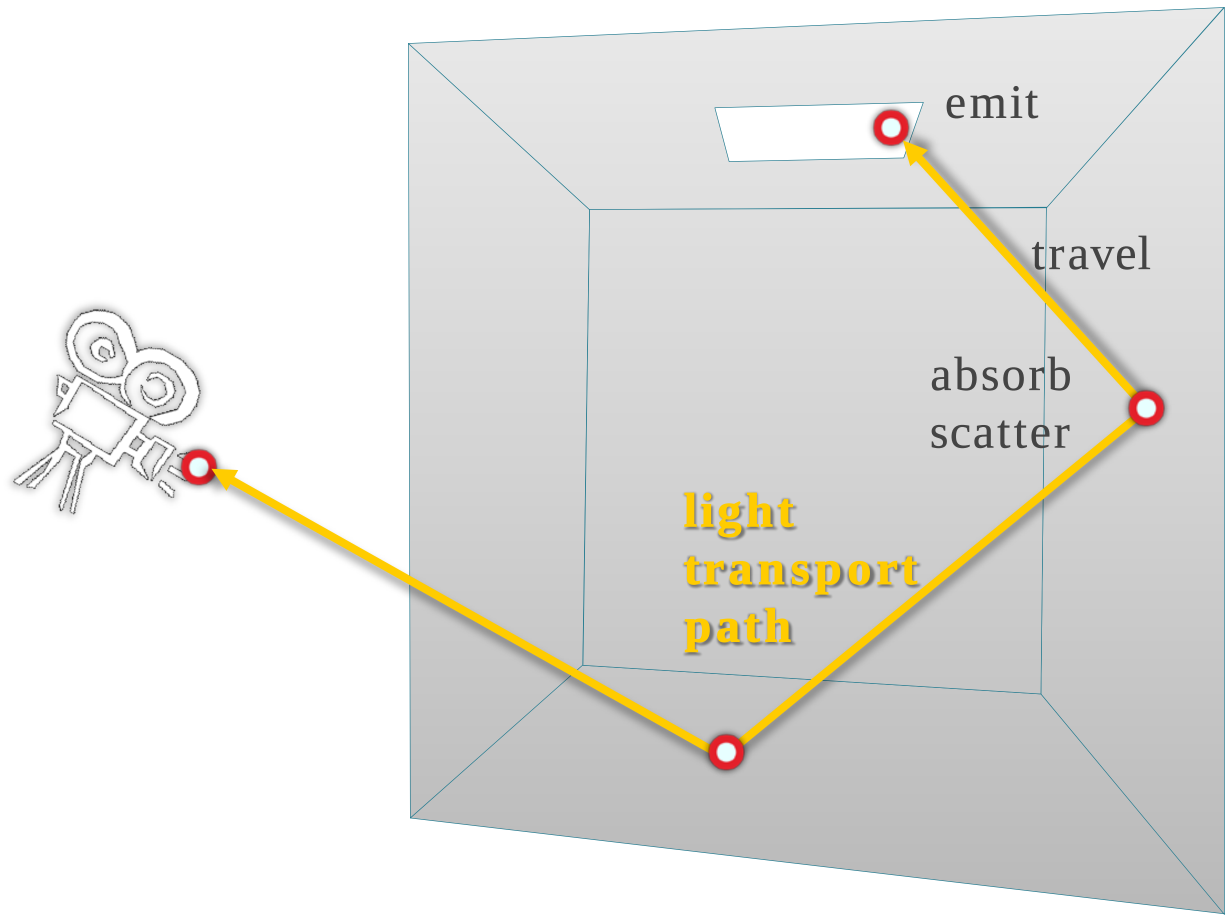
http://cgg.mff.cuni.cz/~jaroslav/

Light transport

- Geometric optics

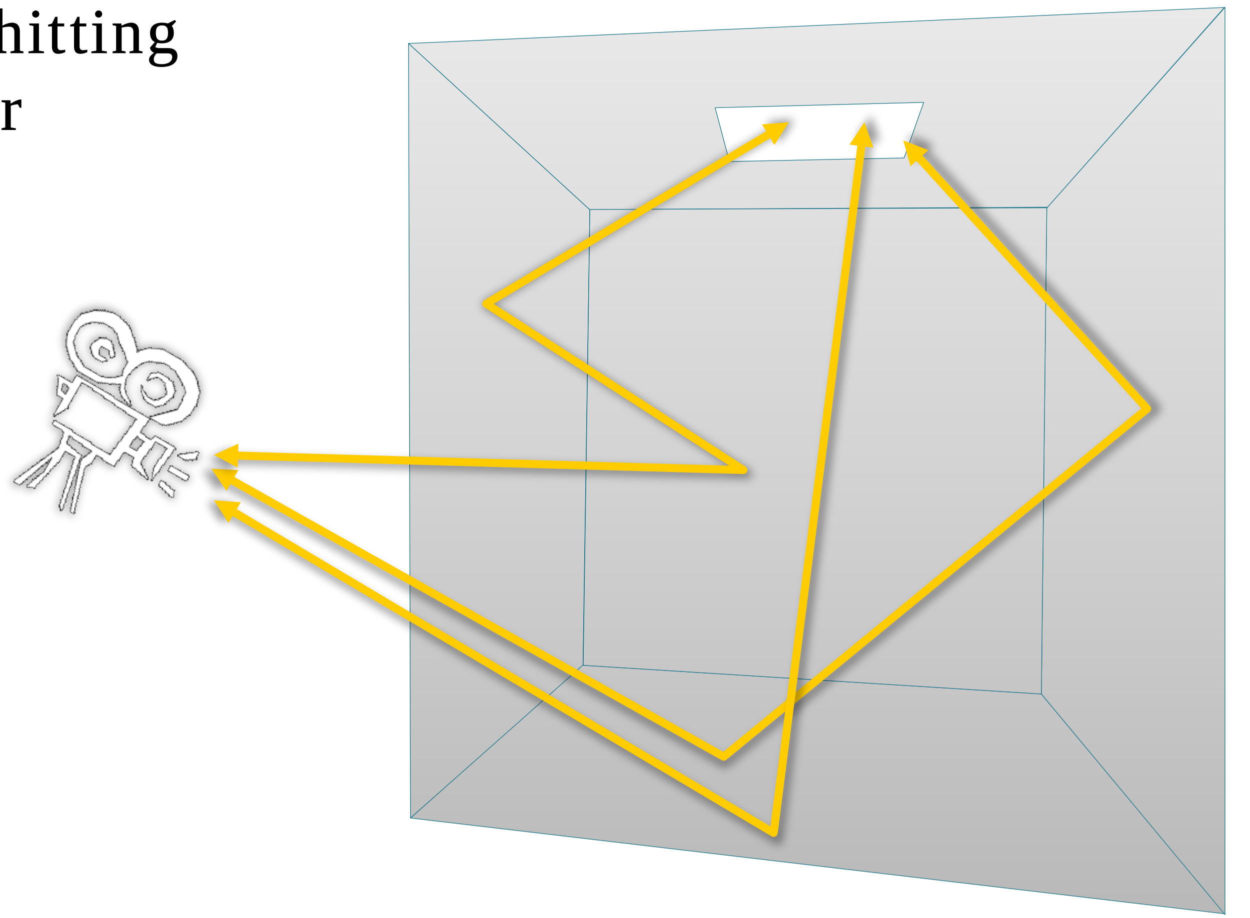


Light transport



Light transport

- **Camera response**
  - all paths hitting the sensor



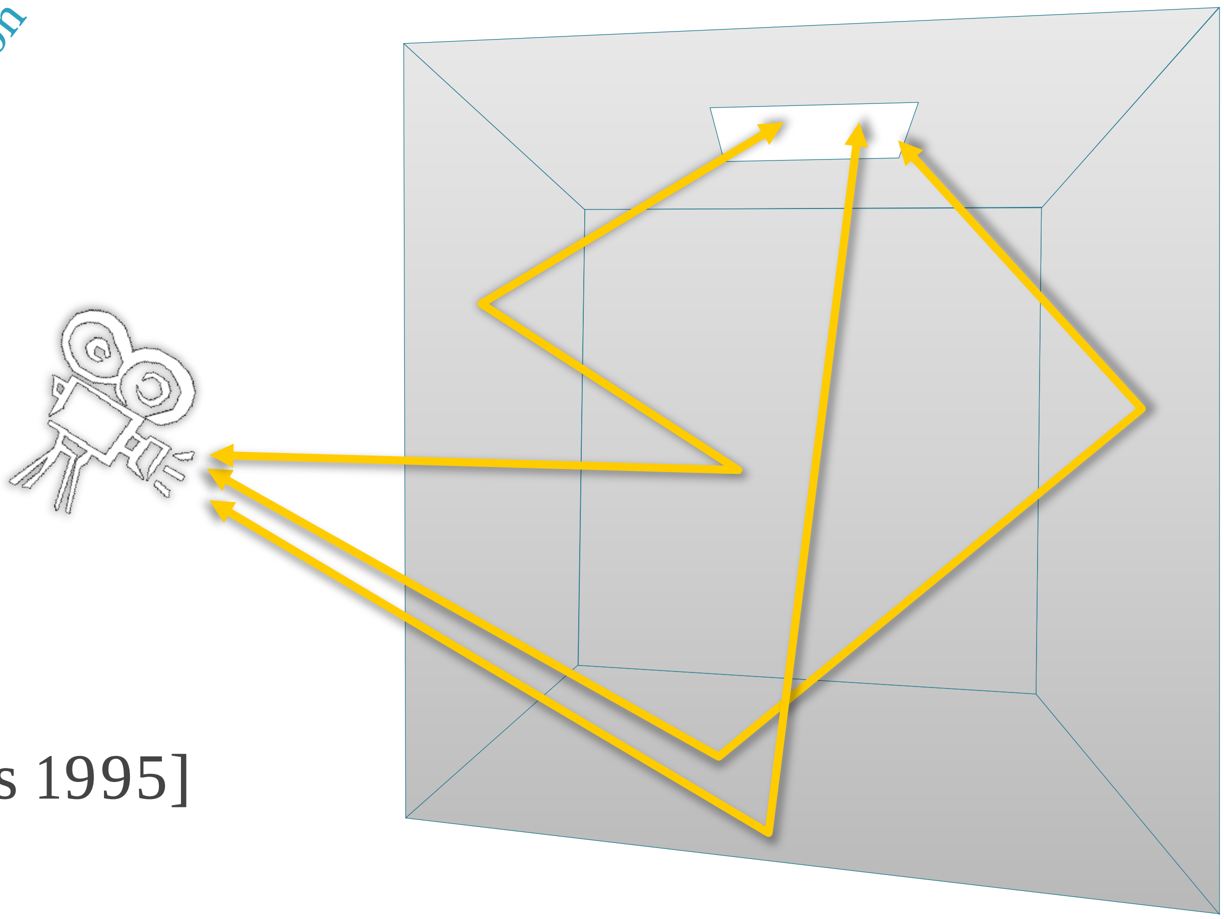
Path integral formulation

$$I_j = \int_\Omega f_j(\bar{x}) \, \mathrm{d}\mu(\bar{x})$$

camera resp. (j-th pixel value)

all paths

measurement contribution function



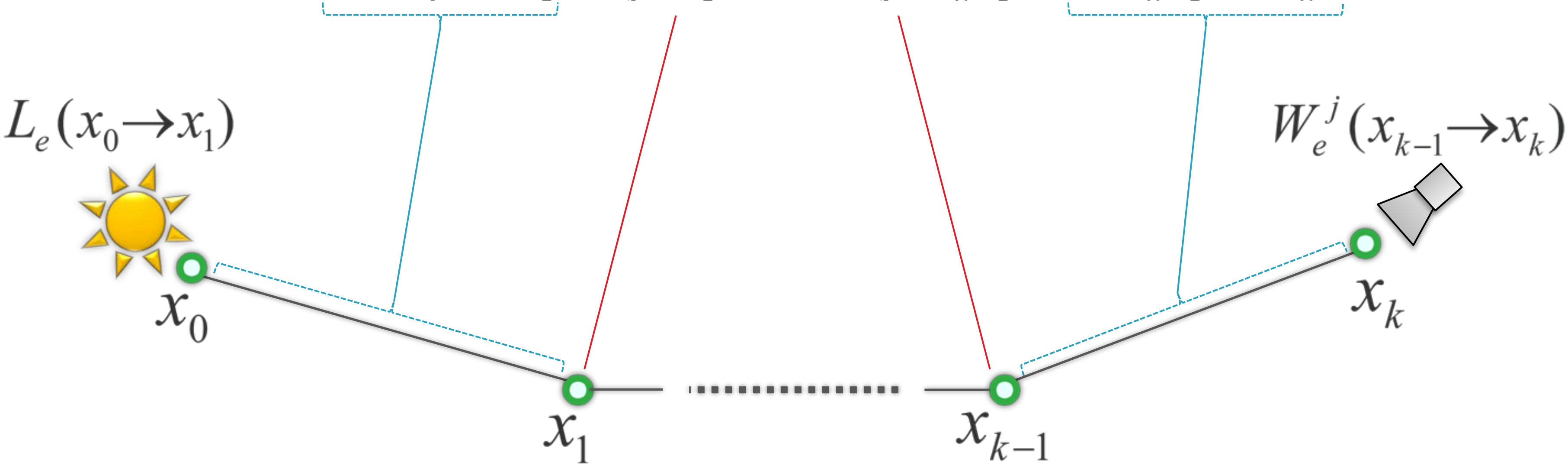
[Veach and Guibas 1995]
[Veach 1997]

Measurement contribution function

$$\bar{x} = x_0 x_1 \ldots x_k$$

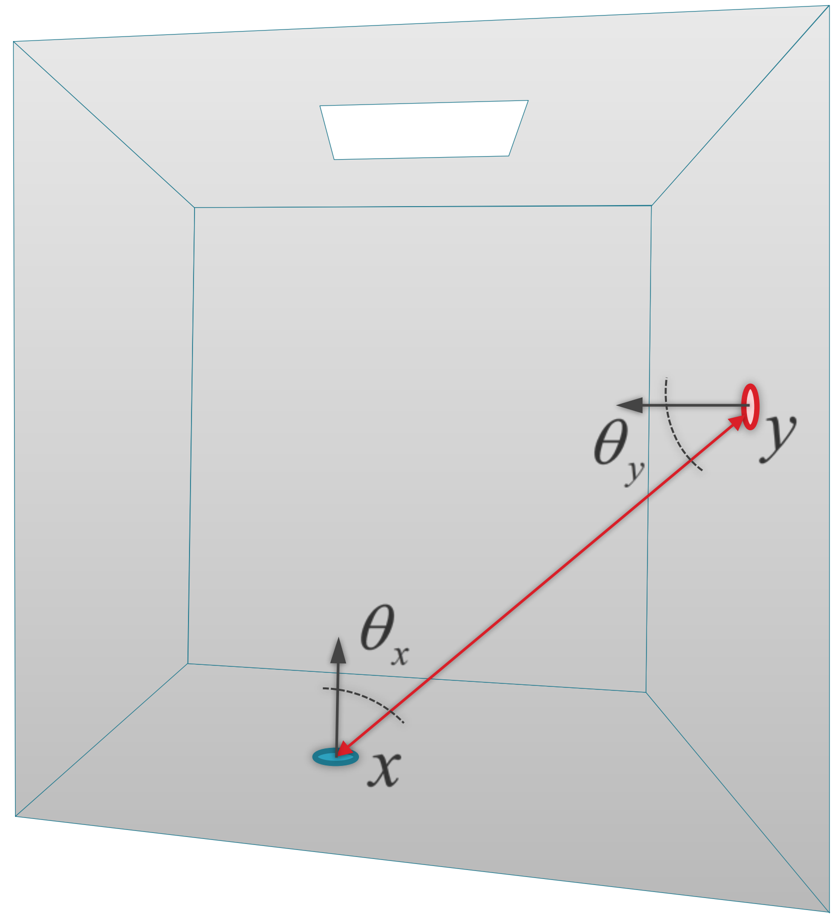
$$f_j(\bar{x}) = \underbrace{L_e(x_0 \rightarrow x_1)}_{\text{emitted radiance}} \; \underbrace{T(\bar{x})}_{\text{path throughput}} \; \underbrace{W_e^j(x_{k-1} \rightarrow x_k)}_{\text{sensor sensitivity ("emitted importance")}}$$

$$T(\bar{x}) = G(x_0 \leftrightarrow x_1)\, \rho_s(x_1) \;\ldots\; \rho_s(x_{k-1})\, G(x_{k-1} \leftrightarrow x_k)$$



Geometry term

$$G(x \leftrightarrow y) = \frac{|\cos\theta_x| \, |\cos\theta_y|}{\|x - y\|^2} V(x \leftrightarrow y)$$



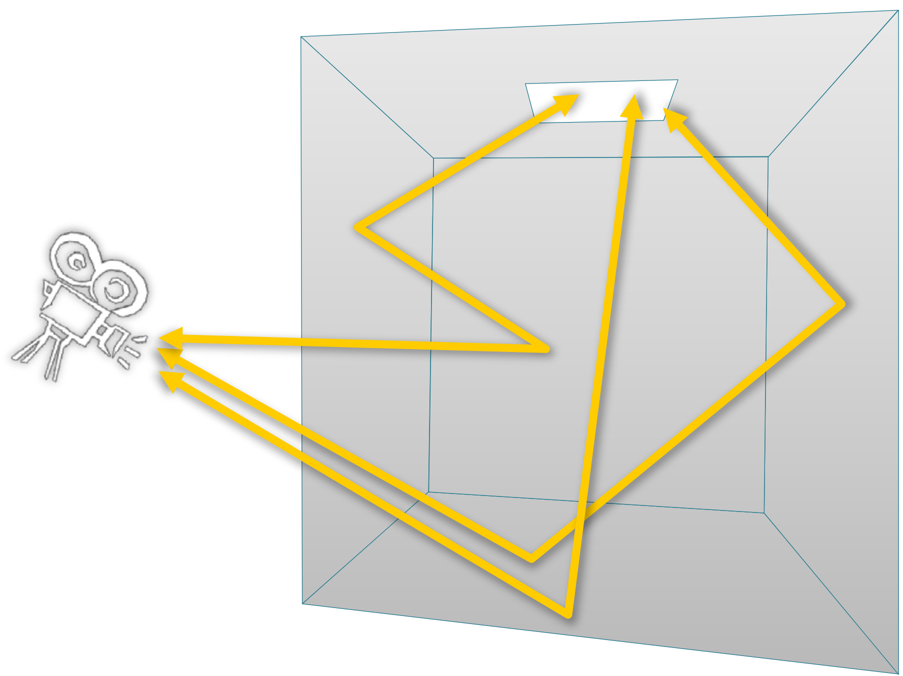
Path integral formulation

$$I_j = \int_{\Omega} f_j(\bar{x}) \, \mathrm{d}\mu(\bar{x})$$

camera resp. (j-th pixel value) ?

all paths

measurement contribution function ✓



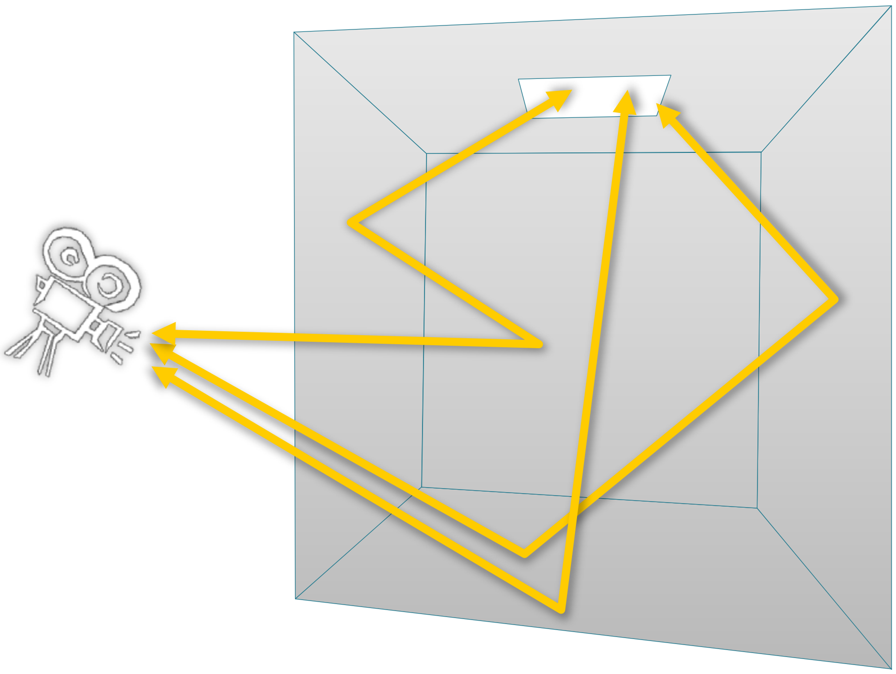
Path integral formulation

$$I_j = \int_{\Omega} f_j(\bar{x})\ \mathrm{d}\mu(\bar{x})$$

$$= \sum_{k=1}^{\infty} \int_{M^{k+1}} f_j(x_0 \ldots x_k)\ \mathrm{d}A(x_0)\ldots\mathrm{d}A(x_k)$$

all path lengths

all possible vertex positions



Path integral

$$I_j = \int_{\Omega} f_j(\bar{x})\, \mathrm{d}\mu(\bar{x})$$

pixel value

all paths

contribution function

RENDERING :

EVALUATING THE PATH INTEGRAL

Path integral

$$I_j = \int_\Omega f_j(\bar{x}) \, \mathrm{d}\mu(\bar{x})$$

pixel value

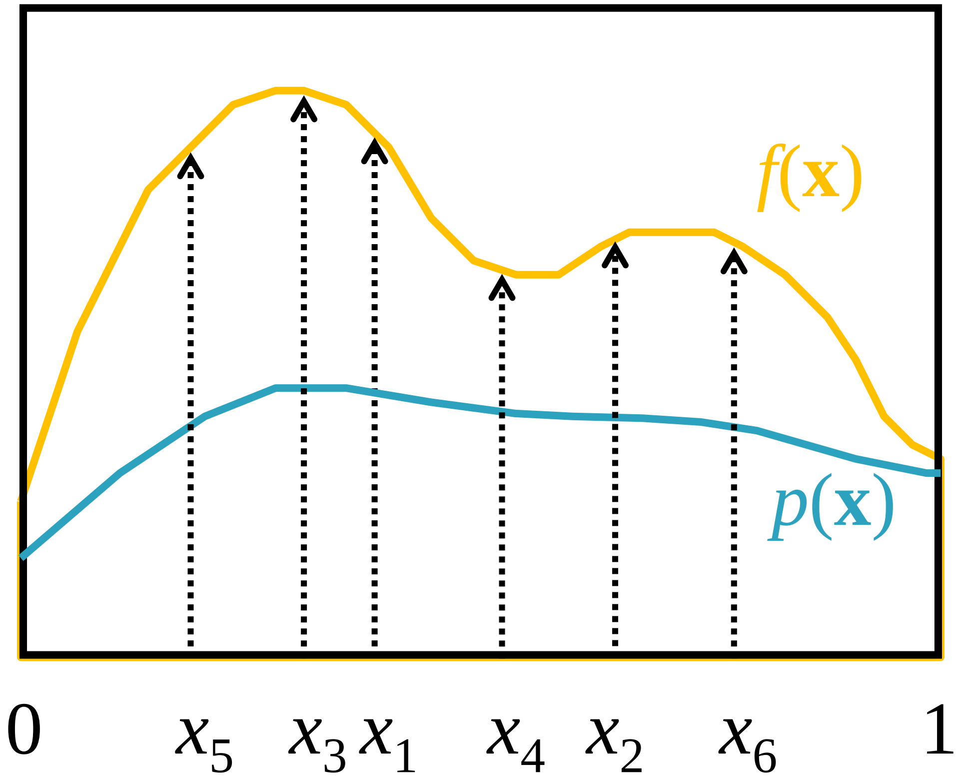
all paths

contribution function

- **Monte Carlo integration**

Monte Carlo integration

- General approach to numerical evaluation of integrals



Integral:

$$I = \int f(x) \mathrm{d}x$$

Monte Carlo estimate of I:

$$\langle I \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}; \quad x_i \propto p(x)$$

Correct „on average“:

$$E[\langle I \rangle] = I$$

MC evaluation of the path integral

Path integral

$$I_j = \int_\Omega f_j(\bar{x})\, \mathrm{d}\mu(\bar{x})$$

MC estimator

$$\langle I_j \rangle = \frac{f_j(\bar{x})}{p(\bar{x})}$$

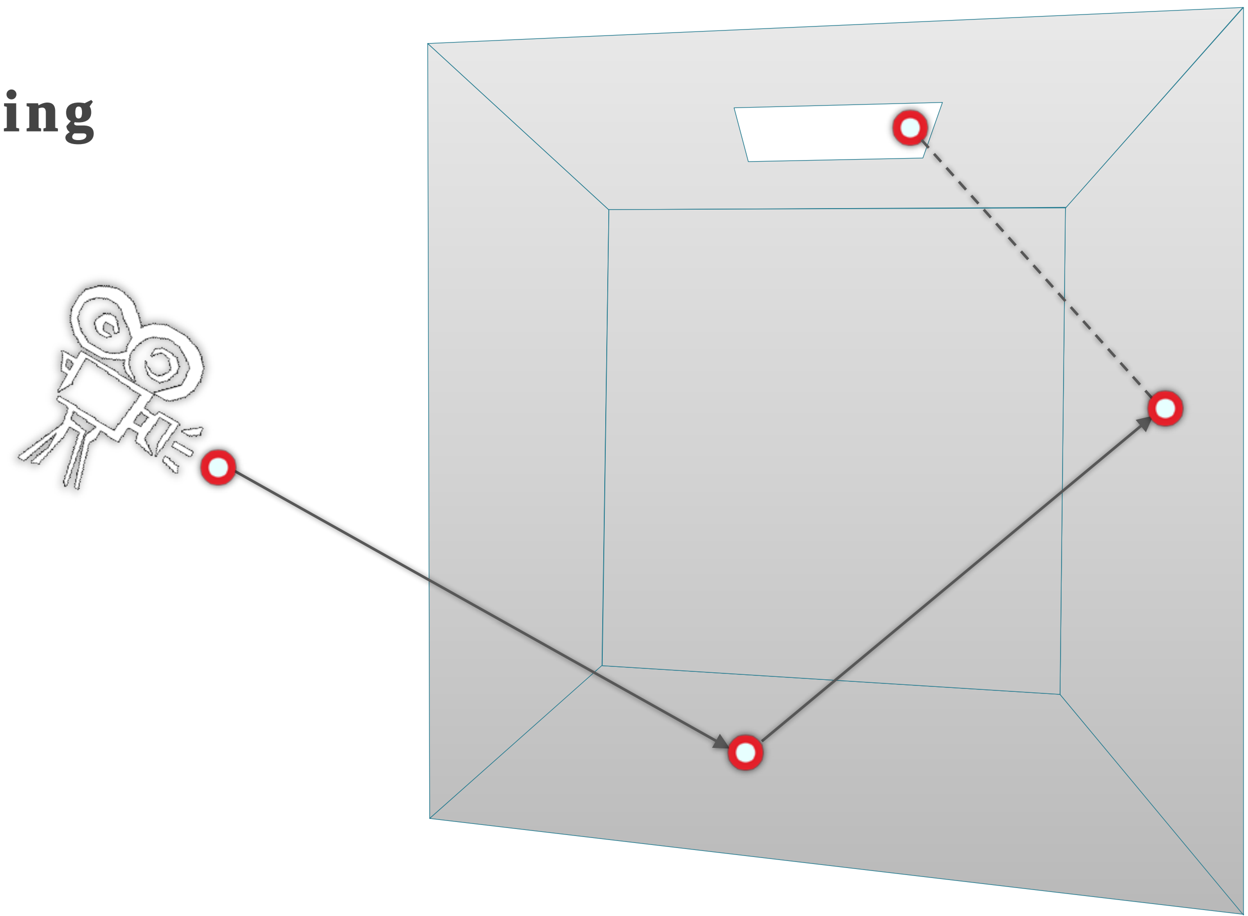
- Sample path $\bar{x}$ from some distribution with PDF $p(\bar{x})$?
- Evaluate the probability density $p(\bar{x})$?
- Evaluate the integrand $f_j(\bar{x})$ ✓

Path sampling

- Algorithms = different path sampling techniques

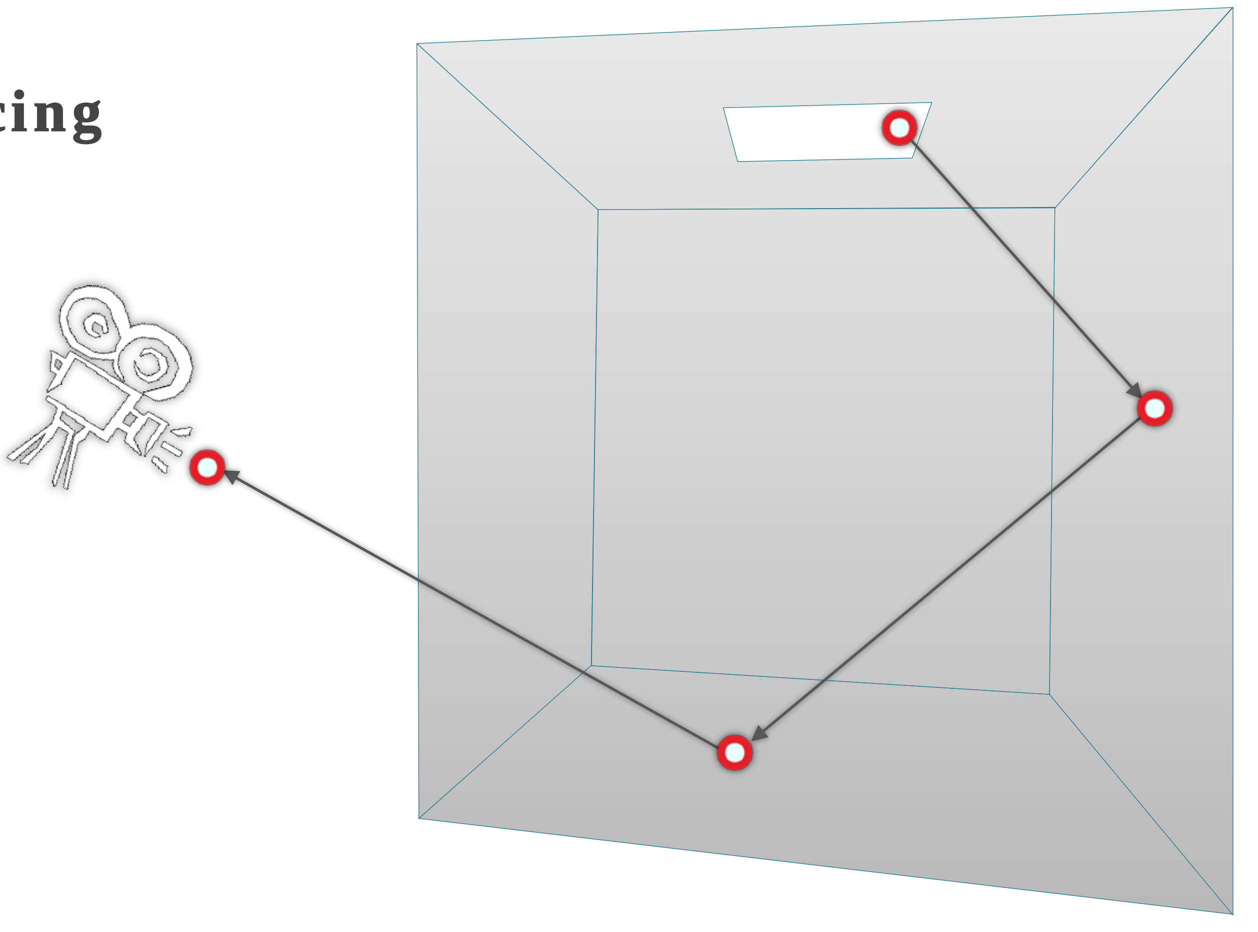
Path sampling

- Algorithms = different path sampling techniques
  - **Path tracing**



Path sampling

- Algorithms = different path sampling techniques
  - **Light tracing**



Path sampling

- Algorithms = different path sampling techniques
- **Same** general form of **estimator**

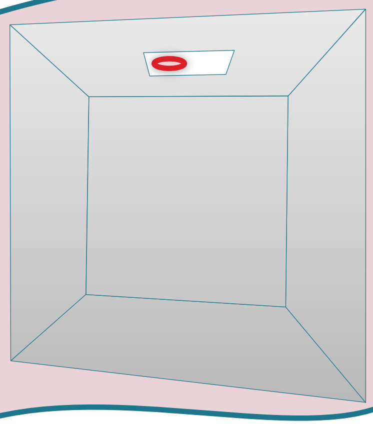
$$\langle I_j \rangle = \frac{f_j(\bar{x})}{p(\bar{x})}$$

- **No importance transport, no adjoint equations!!!**

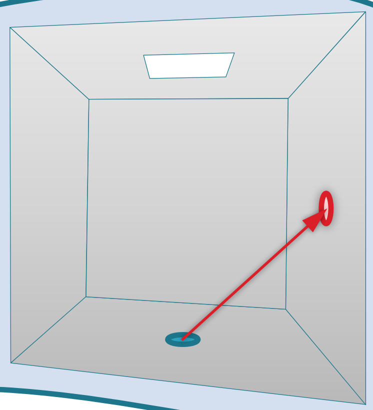
PATH SAMPLING
&
PATH PDF

Local path sampling

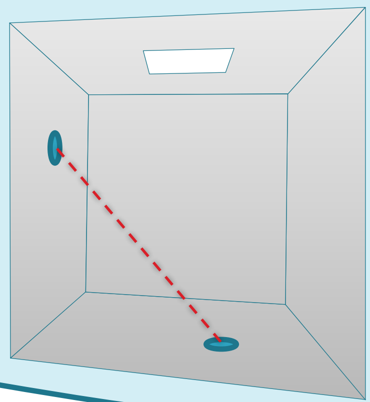
- Sample one path vertex at a time



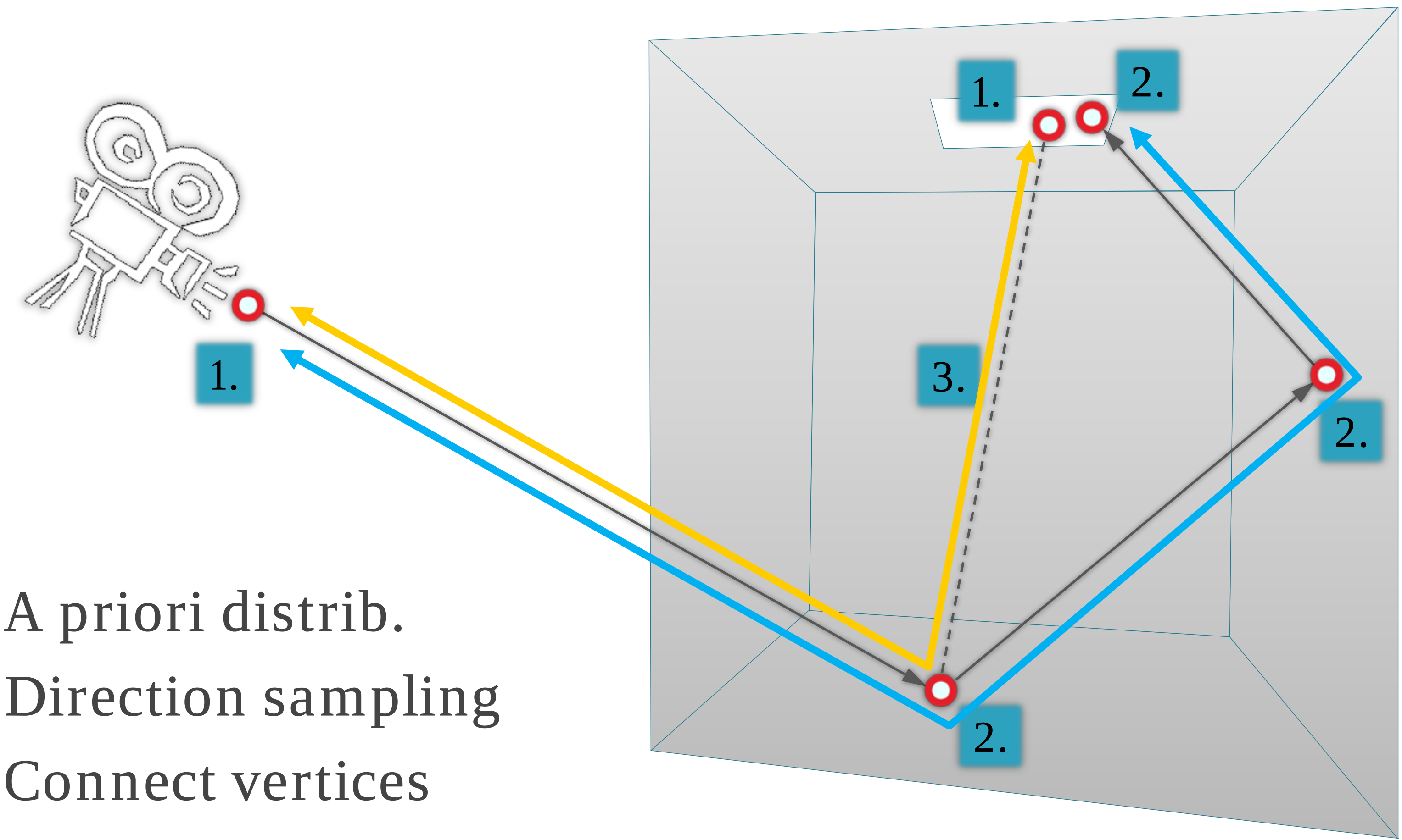
1. From an a priori distribution
   - lights, camera sensors
2. Sample direction from an existing vertex



3. Connect sub-paths
   - test visibility between vertices



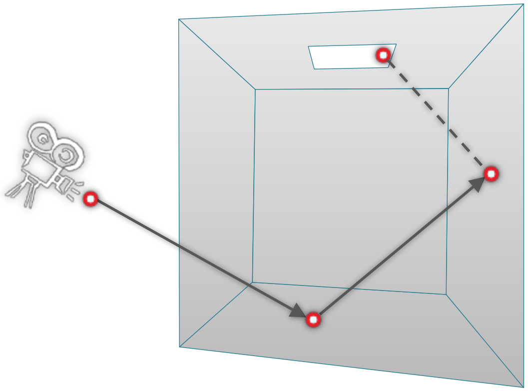
Example – Path tracing



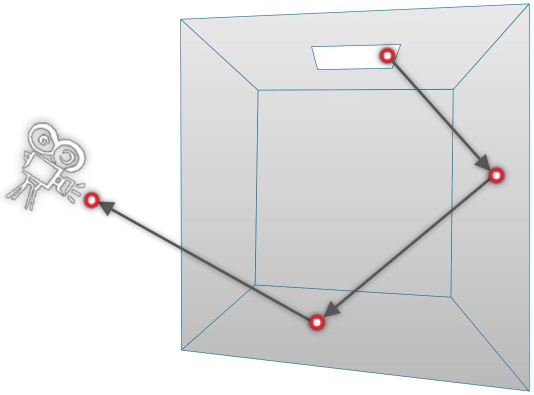
1. A priori distrib.
2. Direction sampling
3. Connect vertices

Use of local path sampling

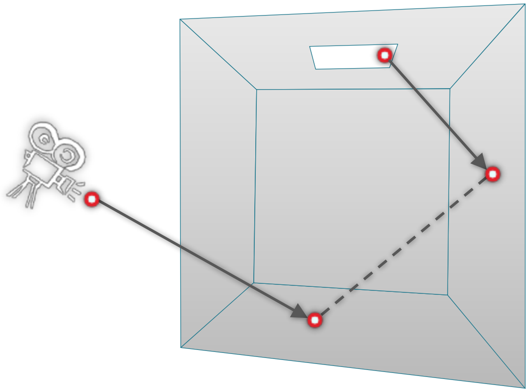
Path tracing



Light tracing



Bidirectional path tracing

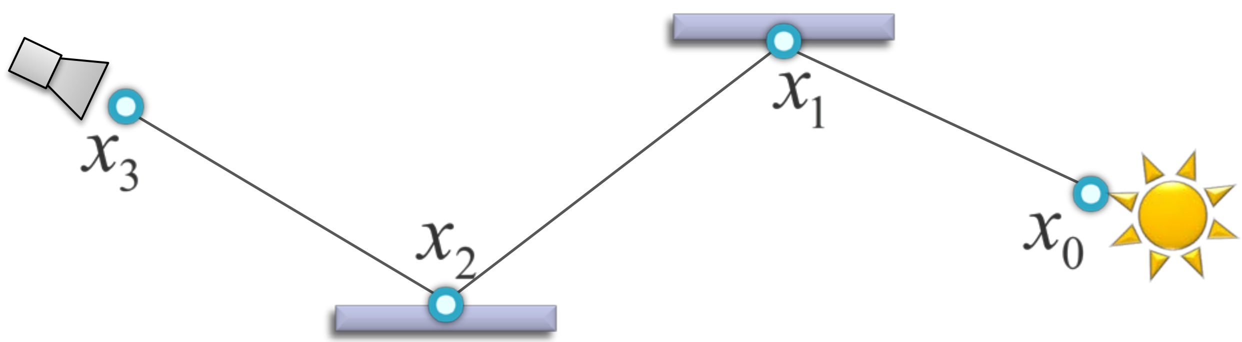


Probability density function (PDF)

path PDF

$$p(\bar{x}) = p(x_0, \ldots, x_k)$$

joint PDF of path vertices

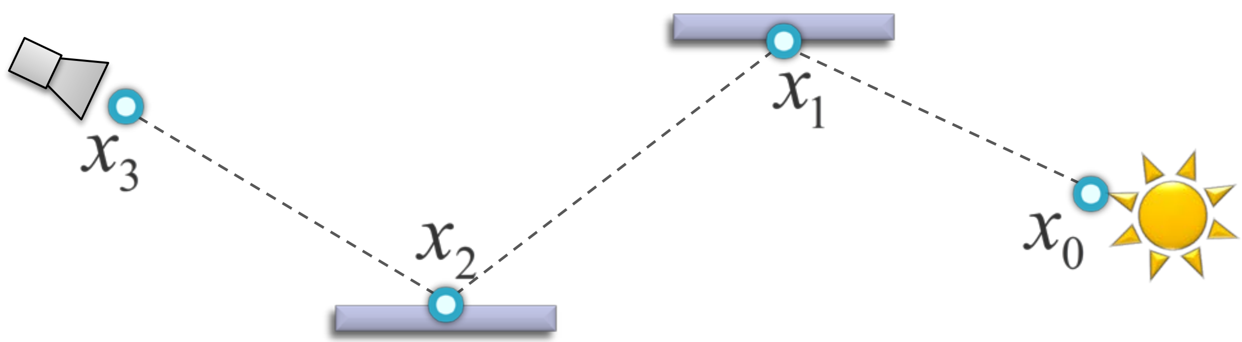


Probability density function (PDF)

path PDF

$$p(\bar{x}) = p(x_0, \ldots, x_k)$$

joint PDF of path vertices



Probability density function (PDF)

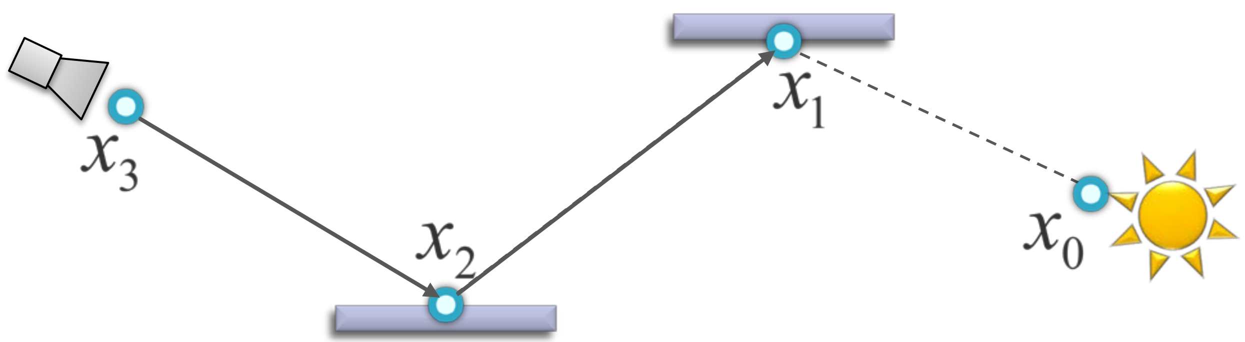
path PDF

$$p(\bar{x}) = p(x_0, \ldots, x_k) = p(x_3)\, p(x_2 \mid x_3)\, p(x_1 \mid x_2)\, p(x_0)$$

joint PDF of path vertices

product of (conditional) vertex PDFs

Path tracing example:



Probability density function (PDF)

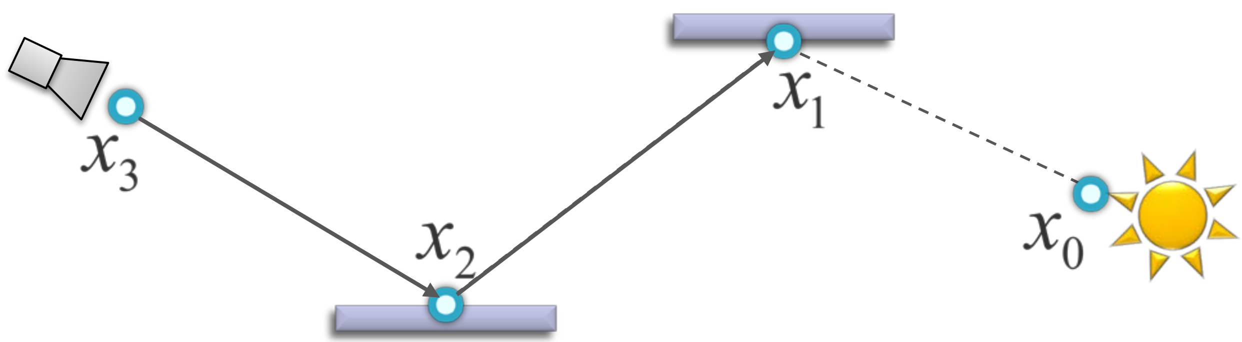
path PDF

$$p(\bar{x}) = p(x_0, \ldots, x_k) = p(x_3)\, p(x_2)\, p(x_1)\, p(x_0)$$

joint PDF of path vertices

product of (conditional) vertex PDFs

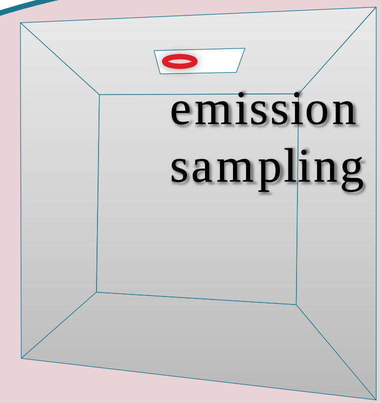
Path tracing example:



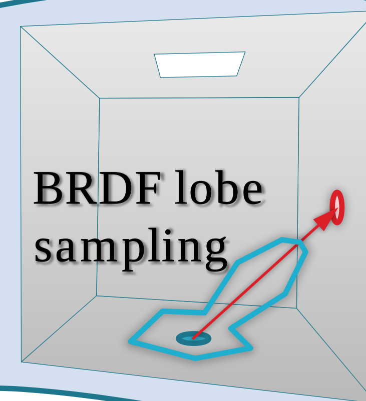
Vertex sampling

- **Importance sampling** principle

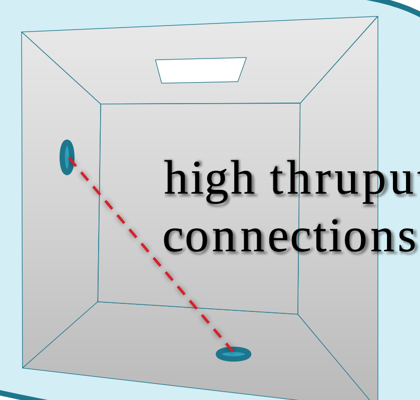
1. Sample from an a priori distrib.



2. Sample direction from an existing vertex

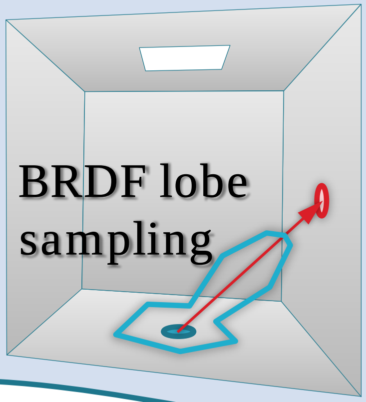


3. Connect sub-paths



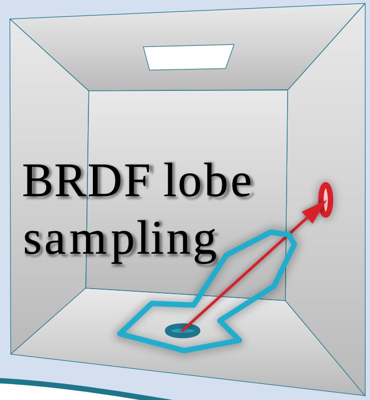
Vertex sampling

- Sample direction from an existing vertex



Measure conversion

- Sample direction from an existing vertex



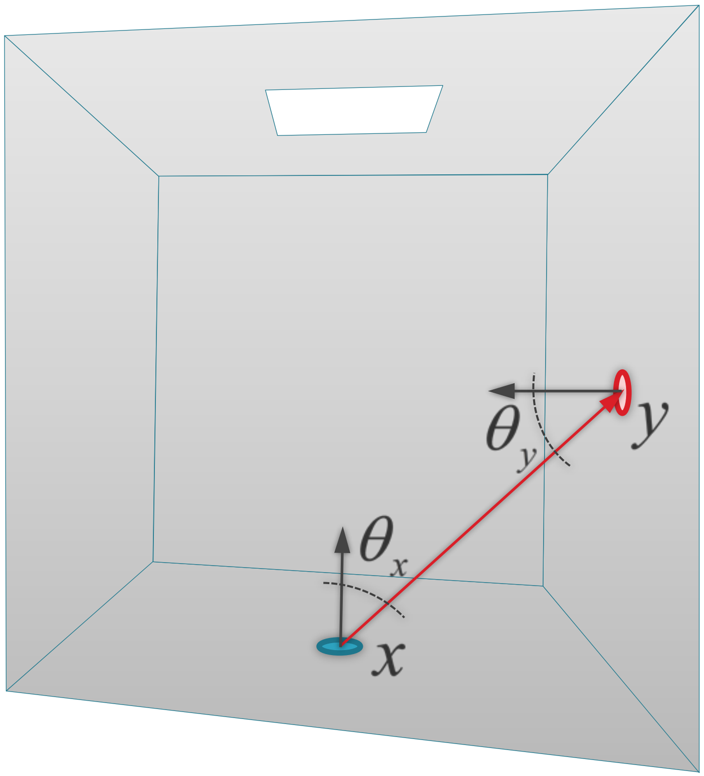
$$p(y) = p^{\perp}(x \rightarrow y)\, G(x \leftrightarrow y)$$

w.r.t. area

w.r.t. proj. solid angle

$$\langle I_j \rangle = \frac{f_j(\bar{x})}{p(\bar{x})}$$

$$= \frac{\cdots \rho_s(x \rightarrow y) G(x \leftrightarrow y) \cdots}{\cdots p^{\perp}(x \rightarrow y) G(x \leftrightarrow y) \cdots}$$



Summary

Path integral

$$I_j = \int_\Omega f_j(\bar{x})\, \mathrm{d}\mu(\bar{x})$$

pixel value

all paths

contribution function

MC estimator

$$\langle I_j \rangle = \frac{f_j(\bar{x})}{p(\bar{x})}$$

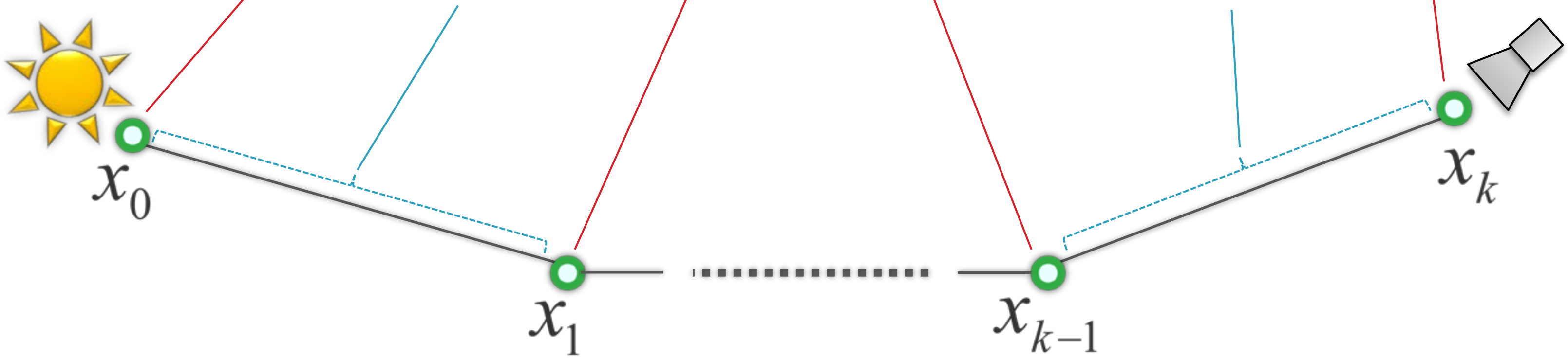
path pdf

sampled path

$$\bar{x} = x_0 \dots x_k$$

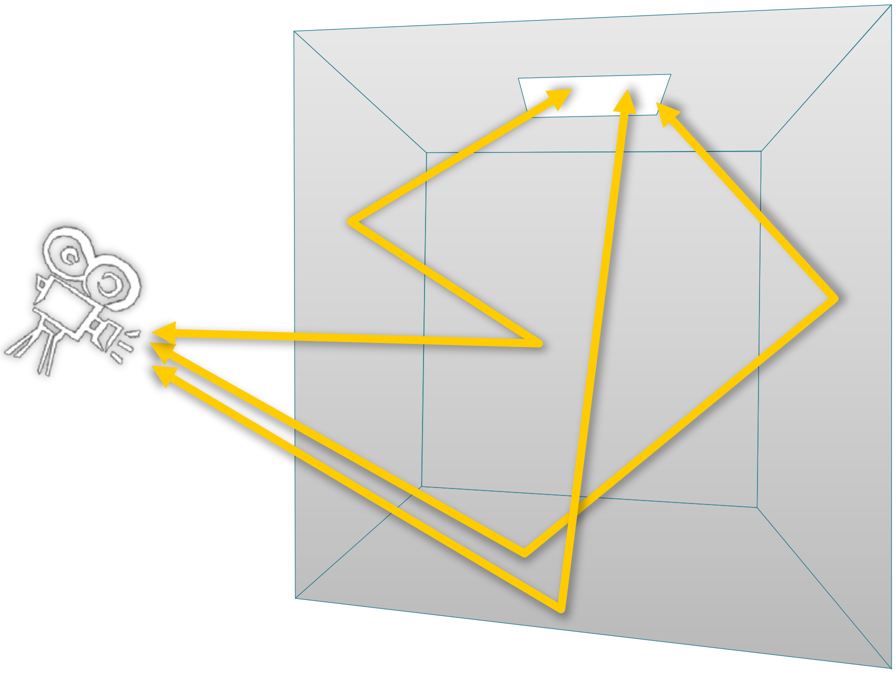
$$p(\bar{x}) = p(x_0) \dots p(x_k)$$

$$f_j(\bar{x}) = L_e\, G(x_0 \leftrightarrow x_1)\, \rho_s(x_1) \dots \rho_s(x_{k-1})\, G(x_{k-1} \leftrightarrow x_k)\, W_e^j$$



Summary

- **Algorithms**
  - different path sampling techniques
  - different path PDF



Time for questions...